Probability Questions

1. In a group of 40 people, 10 are healthy and every person of the remaining 30 has either high blood pressure, a high level of cholesterol or both. If 15 have high blood pressure and 25 have high level of cholesterol,   
   a) how many people have high blood pressure and a high level of cholesterol?   
   If a person is selected randomly from this group, what is the probability that he/she   
   b) has high blood pressure (event A)?   
   c) has high level of cholesterol(event B)?   
   d) has high blood pressure and high level of cholesterol (event A and B)?   
   e) has either high blood pressure or high level of cholesterol (event A or B)?   
   f) Use the above to check the probability formula: P(A or B) = P(A) + P(B) - P(A and B).
2. A committee of 5 people is to be formed randomly from a group of 10 women and 6 men. Find the probability that the committee has   
   a) 3 women and 2 men.   
   b) 4 women and 1 men.   
   c) 5 women.   
   d) at least 3 women.

There are 2 solution paths:

* 1. Calculate the number of combinations of draws:
     1. 10C­3\*6C2 / ­16C5 == P(A)
  2. As all combinations occur with equal probability calculate probability of one event then multiply it by combinations of the chosen 5.

P(a) = 10/16 \* 9/15 \* 8/14 \* 6/13 \* 5/12

5C­2 == 5C3 == 10

P(A) = p(a) \* 5C­2

1. In a school, 60% of pupils have access to the internet at home. A group of 8 students is chosen at random. Find the probability that   
   a) exactly 5 have access to the internet.

b) at least 6 students have access to the internet.

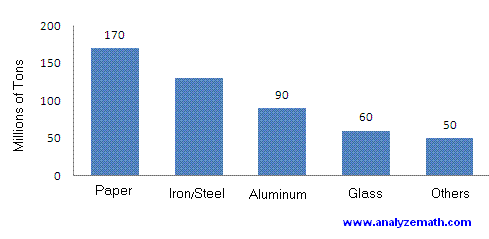
1. The grades of a group of 1000 students in an exam are normally distributed with a mean of 70 and a standard deviation of 10. A student from this group is selected randomly.   
   a) Find the probability that his/her grade is greater than 80.

Using the cdf: 1 – p(x < 80)   
b) Find the probability that his/her grade is less than 50.

1 – p(x < 80) or 50%  
c) Find the probability that his/her grade is between 50 and 80.

P(x < 80) – p(x < 50)  
d) Approximately, how many students have grades greater than 80?

1000 \* (1 – p(x < 80))

1. in a certain country last year a total of 500 million tons of trash was recycled. The chart below shows the distribution, in millions of tons, for the different types of trashes.   
      
   a) How many tons of Iron/Steel was recycled?   
   b) What percent of the recycled trash was glass?

1.    
  
a) Let x be the number of people with both high blood pressure and high level of cholesterol. Hence (15 - x) will be the number of people with high blood pressure ONLY and (25 - x) will be the number of people with high level of cholesterol ONLY. We now express the fact that the total number of people with high blood pressure only, with high level of cholesterol only and with both is equal to 30.   
(15 - x) + (25 - x) + x = 30   
solve for x: x = 10   
b) 15 have high blood pressure,hence P(A) = 15/40 = 0.375   
c) 25 have high level of cholesterol, hence P(B) = 25/40 = 0.625   
d) 10 have both,hence P(A and B) = 10/40 = 0.25   
e) 30 have either, hence P(A or B) = 30/40 = 0.75   
e) P(A) + P(B) - P(A and B) = 0.375 + 0.625 - 0.25 = 0.75 = P(A or B)

2   
a) In what follows nCr = n! / [ (n - r)!r! ] and is the number of combinations of n objects taken r at the time and P(A) is the probability that even A happens.   
There are 16C5 ways to select 5 people (committee members) out of a total of 16 people (men and women)   
There are 10C3 ways to select 3 women out of 10.   
There are 6C2 ways to select 2 men out of 6.   
There are 10C3\*6C2 ways to select 3 women out of 10 AND 2 men out of 6.   
P(3 women AND 2 men) = 10C3\*6C2 / 16C5 = 0.412087   
b) similarly: P(4 women AND 1 men) = 10C4\*6C1 / 16C5 = 0.288461   
c) similarly: P(5 women ) = 10C5\*6C0 / 16C5 = 0.0576923 (in 6C0 the 0 is for no men)   
d) P(at least 3 women) = P(3 women or 4 women or 5 women)   
  
since the events "3 women" , "4 women" and "5 women" are all mutually exclusive, then   
P(at least 3 women) = P(3 women or 4 women or 5 women) = P(3 women) + P(4 women) + P(5 women) = 0.412087 + 0.288461 + 0.0576923 = 0.758240

3   
a) If a pupil is selected at random and asked if he/she has an internet connection at home, the answer would be yes or no and therefore it is a binomial experiment. The probability of the student answering yes is 60% = 0.6. Let X be the number of students answering yes when 8 students are selected at random and asked the same question. The probability that X = 5 is given by the binomial probability formula as follows:   
P(X = 5) = 8C5 (0.6)5 (1-0.6)3 = 0.278691   
b) P(X ≥ 6) = P(X = 6 or X = 7 or X = 8)   
Since all the events X = 6, X = 7 and X = 8 are mutually exclusive, then   
P(X ≥ 6) = P(X = 6) + P(x = 7) + P(X = 8)   
= 8C6 (0.6)6 (1-0.6)2 + 8C7 (0.6)7 (1-0.6)1 + 8C8 (0.6)8 (1-0.6)0  
= 0.315394

4   
a) x = 80 , z = (80 - 70)/10 = 1   
Probablity for grade to be greater than 80 = 1 - 0.8413 = 0.1587   
b) x = 50 , z = (50 - 70)/10 = -2   
Probablity for grade to be less than 50 = 0.0228   
c) The z-scores for x = 50 and x = 80 have already been calculated above.   
Probablity for grade to be between 50 and 80 = 0.8413 - 0.0228 = 0.8185   
d) 0.1587 \* 1000 = 159 (rounded to the nearest unit)

5   
a) 500 - (170+90+60+50) = 130 tons of steel/iron was recycled.   
b) 60/500 = 0.12 = 12% of the total recycled was glass.

When a die is rolled and a coin (with Heads and Tails) is tossed, find the probability of obtaining   
a) Tails and an even number,

P(T) = 1/2, P(Even) = ½ 🡪 P(T & Event) = 1/4  
b) a number greater 3,

P(n > 3) = ½   
c) Heads or an odd number,

P(H) + P(Odd) – P(H & Odd) = ½ + ½ - ¼ = ¾

A committee of 6 people is to be formed from a group of 20 people. The committee has to have the number of women double that of the men. In how many ways can this committee be formed if there are 12 men?

* Need 4 women, 2 men from 8 women, 12 men
* P(4 Women & 2 Men) =
* Can choose 4 women & 2 men in ways (4620)

Denis examined the records of the clients who had joined his gym more than six months ago, and he found these statistics:

*P*(joined in January)=0.12

*P*(member for over 6 months)=0.5

*P*(over 6 months and January)=0.024​

**Find the probability that a client remained a member for more than 66 months, given that the client joined in January.**

*P*(over 6 months ∣ January)=

* P(A|B) = P(A and B) / P(B)
* *P*(over 6 months ∣ January)= .024 / .12

P(A)

P(B)

P(A & B)

Question on the Binomial distribution:

**when the sample size n is increased, what happens to P(X** ≥ **mean)? If the sample size became very large what would you expect P(X** ≥ **mean) to approach? Briefly explain your answer.**

* As the sample size increase the binomial approaches the normal in distribution. This implies. For small n, P(X>= mean) > .5. As n increases this will approach .5 as the distributions converge with µ = n\*p. It is then expected that P(x >= µ) = .5